Introduction to Probability
For Students

Performance Expectations (CCSS)

This lesson addresses the following Common Core State Standard (CCSS) for Grade 7:
- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

When you hear that there is a 30% chance of rain tomorrrow, what does this mean? It means that days with weather like today are followed by days with rain 30% of the time. We say that 30% is the *probability* that it will rain. Another way to say this is that about $\frac{30}{100}$ or $\frac{\square}{10}$ of the time it will rain tomorrow.

The **probability** of something is the fraction of the chances that it will happen. It is a way of saying what may happen in the future, or predicting the future.

Predicting the rain is complicated. Let us look at something simpler. If you toss a coin, you will get heads or tails as a result. There are two possible results. One of those two results is heads. We predict that if you toss the coin many times, you will get heads 1 out of 2 times or $\frac{1}{2}$ of the time. The probability of heads is $\frac{1}{2}$. So if you toss the coin 100 times, you should get about _____ heads. Of course, you can get other numbers of heads. The result is uncertain. Probability has to do with things that are uncertain.

Here are some basic definitions so we can give a proper definition of probability and study it more carefully.

The result of an action is **random** when you cannot say the next thing that will happen.

An **experiment** is an action that has different random results that you can name. Tossing a coin is an experiment. The results are heads or tails. Rolling a number cube is an experiment. The results are 1, 2, 3, 4, 5, or 6. Looking at the weather tomorrow is an experiment (that has many results, but in the first paragraph the results were rain or no rain).

A **trial** is doing the experiment one time.

An **outcome** is one possible result of an experiment. Rolling the number 4 is an outcome of rolling a number cube.

The **sample space** is the set of all possible results or outcomes of an experiment. The sample space for rolling a number cube is the set {1, 2, 3, 4, 5, 6}. This sample space has __ outcomes.

An **event** is a description of several possible results or outcomes of an experiment. "Rolling an odd number" is an event for rolling a number cube. It describes rolling the number 1 or 3 or 5. This event has __ outcomes. An event could have just one outcome.

The **probability of an event** is the fraction

$$\frac{\text{the number of outcomes in the event}}{\text{the number of outcomes in the sample space}}$$

For example, if the experiment is rolling a number cube, and the event is "rolling an odd number," then the probability is $\frac{3}{6}$ or $\frac{1}{2}$. Probability can be written as a fraction, as a decimal (0.5), or as a percent (____%).
It is just as likely to get an even number as an odd number when you roll a number cube.

We say that an event with probability $\frac{1}{2}$ is neither likely nor unlikely.

Suppose you bought a raffle ticket and 100 tickets were sold. One ticket is chosen at random from a bowl.
What is the probability that you will win the raffle? The event "you winning the raffle" has one outcome (your ticket is chosen from the bowl). The sample space has 100 outcomes (there are 100 different tickets that could be chosen). So the probability of you winning is the fraction $\frac{1}{100}$. Note that this is a number close to 0, and that it is unlikely for you to win this raffle. In general, if the probability of an event is a number close to 0, this means that the event is unlikely to happen.

Suppose you bought 2 of the tickets. Winning is now a more likely event and the probability is now $\frac{\square}{100}$. This fraction is greater than $\frac{1}{100}$. In general, the greater the probability (fraction) the more likely the event is.

Example 1

Two coins are tossed simultaneously. What is the sample space? What is the probability of getting two tails?

Solution:

Let H represent heads on one coin and T represent tails on one coin. The sample space is

$$HH \quad HT \quad TH \quad TT$$

The sample space has __ outcomes. The event "two tails" has one outcome (TT). The probability is $\frac{1}{4}$.

Example 2

Two coins are tossed simultaneously. What is the probability of getting at least one tail?

Solution:

This has the same sample space as in the previous example. The event has 3 outcomes: HT, TH, and TT. The probability is $\frac{3}{\Box}$.

Example 3

A standard deck of cards has 4 "suits:" hearts, spades, clubs, and diamonds. For each suit, there are 13 cards labeled 2 thru 10, plus jack, queen, king, and ace. So the deck has 52 cards all together. One card is picked at random. What is the probability that the card is an ace?

Solution:

There are 4 outcomes in the event: ace of hearts, spades, clubs, or diamonds. There are

____ outcomes in the sample space. The probability is $\frac{4}{52}$ or $\frac{\Box}{13}$.

Going back to the raffle, suppose you bought 99 of the tickets. It is very likely that you would win. The probability is $\frac{99}{100}$. This is a number that is close to 1. In general, a probability close to 1 means that an event is very likely.

Let us examine the definition of the probability of an event again:

$$\frac{\text{the number of outcomes in the event}}{\text{the number of outcomes in the sample space}}$$

The denominator counts all the things that can happen. The numerator counts some (maybe all) of the things that can happen. So the numerator is less than or equal to the denominator. Thus, the value of the fraction is at most 1. Because you are counting

things, you cannot get a negative number. So the value of the fraction is at least 0. Thus, probability is a number between 0 and 1.

The probability of an event can be exactly equal to 0 or 1. If you roll a number cube labeled 1 through 6, it is impossible to get a 7. You can say that the number of ways to get a 7 is 0. So the probability of "getting a 7" is $\frac{0}{6}$ or 0. If you roll a number cube labeled 1 through 6, it is certain that you will get a number less than 7. The number of outcomes in this event is 6 (all the numbers from 1 to 6). Thus, the probability of "getting less than 7" is $\frac{6}{6}$ or 1.

In general, a probability of 0 means the event is impossible and a probability of 1 means the event is certain to happen.

Exercises

Match the descriptions to the vocabulary words.

1. one possible result
2. the set of all possible results   event
3. when you cannot say the next thing   experiment
that will happen   outcome
4. an action that has different random   random
results that you can name   sample space
5. doing the experiment one time   trial
6. a description of several possible
results or outcomes

For each experiment, write the sample space.

7. Picking a ball out of a box with a red, orange, yellow, and green ball

8. Tossing three coins simultaneously

9. Rolling two number cubes, each labeled 1 through 6

Fill in the blanks. Each blank could represent a word, a phrase, or a number.

10. Probability of an event is defined to be the number of _____ over the number of
_____.

11. The least value for the probability of an event is _____.
The greatest value is _____.

12. A probability of 0 means the event is _____.
A probability near 0 means the event is _____.

13. A probability of 1 means the event is _____.
A probability near 1 means the event is _____.

Find the probability of each event.

14. Experiment: Pick a card at random from a standard deck of 52 cards.
Event: Picking a diamond

15. Experiment: A box has a green, orange, and purple ball and you pick one at random.

Event: Picking a purple ball

16. Experiment: Flip two coins.
Event: Getting exactly one head

17. Experiment: Roll one number cube.
Event: Getting at least 3

18. Experiment: Roll a pair of number cubes.
Event: Getting the same number on the two cubes

19. Experiment: Roll a pair of number cubes.
Event: Getting a sum of 9

20. Experiment: 100 raffle tickets are sold and you buy all of them. One is picked at random.
Event: You win the raffle.

21. Experiment: 100 raffle tickets are sold and you do not buy a ticket. One is picked at random.
Event: You win the raffle.

22. You step into an elevator on the first floor of a 10-story building. A random person steps on after you. What is the probability that they are going to the same floor as you?

23. You step off the elevator in an office building that you have never been to. You are in the middle of a hallway and can go left or right. At both ends of the hall there are halls where you can choose to go left or right. If you choose randomly, and there is only one way to get to your destination, what is the probability you will get there on your first try?