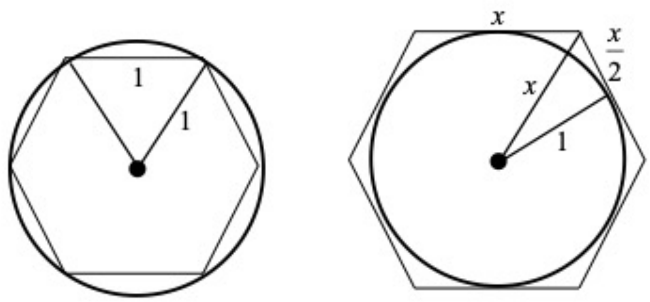
Enrichment Material
Approximation of π

You have been told that $\pi \approx 3.14$, but have not been shown where this number comes from. Here you will be shown a simple proof that $3 < \pi < 2\sqrt{3} \approx 3.464$. A similar, but more complicated, procedure could be used to get a better approximation.

A circle of radius 1 is shown with inscribed and circumscribed regular hexagons. A regular hexagon can be divided into equilateral triangles, partially indicated by the 1's and the x's in the two circles.



The basic idea is that the circumference of the circle is greater than the perimeter of the inscribed hexagon and less than the perimeter of the circumscribed hexagon.

The circumference of the circle is 2π and the perimeter of the inscribed hexagon is 6. So

$$6 < 2\pi \Rightarrow 3 < \pi\,.$$

Let x represent the side length of the circumscribed hexagon. Then its perimeter is $6x$. The right triangle shown is half of an equilateral triangle and that is why one of the legs is $x/2$. Using the Pythagorean theorem

$$1^2 + \left(\frac{x}{2}\right)^2 = x^2$$

you can solve the equation to obtain $x = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$.

Compare the circumference of the circle to the perimeter of the circumscribed hexagon:

$$2\pi < 6x$$

$$\pi < 3x$$

$$\pi < 2\sqrt{3}$$

Exercise: Provide the missing steps in solving the equation.